Resultant of Two S.H.M.s AT RIGHT ANGLES

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Remaining part of Important case:-

case (ii) when $\phi = \frac{\pi}{4}$, then

equn $\frac{y^2}{b^2} - \frac{2xy}{ab}\cos\phi + \frac{x^2}{a^2} = \sin^2\phi$ —③

become

$$\frac{y^2}{b^2} - \frac{2xy}{ab}\cdot\frac{1}{\sqrt{2}} + \frac{x^2}{a^2} = \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\frac{y^2}{b^2} - \frac{\sqrt{2}xy}{ab} + \frac{x^2}{a^2} = \frac{1}{2}$$

This represents an oblique ellipse as shown in fig ②

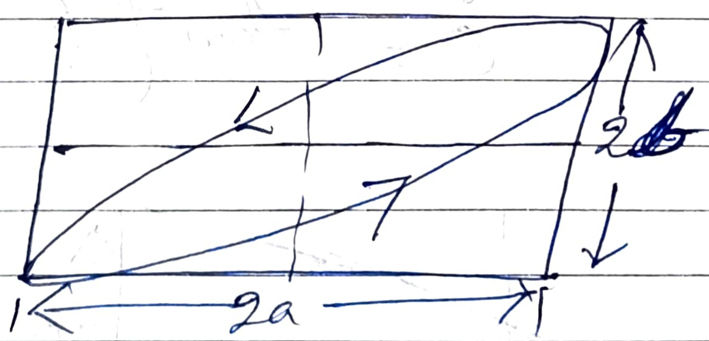


fig :- ②

Case (iii) when $\phi = \frac{\pi}{2}$

equ^n three become

$$\frac{y^2}{b^2} - \frac{2xy}{ab}\cos 90^\circ + \frac{x^2}{a^2} \sin^2 90^\circ$$

$$\frac{y^2}{b^2} + \frac{x^2}{a^2} = \sin^2 90^\circ = 1$$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

This represents an ellipse whose major & minor axes coincide with the co-ordinate axes. like fig ③

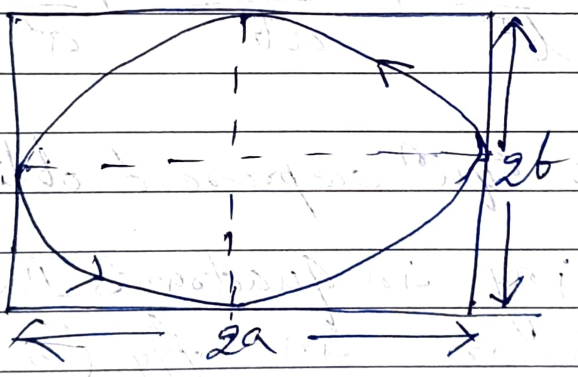


fig 3

Special case If $a = b$ then

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$$

$$\Rightarrow x^2 + y^2 = a^2$$

This represents a ~~Circle~~ Circular as fig. ④

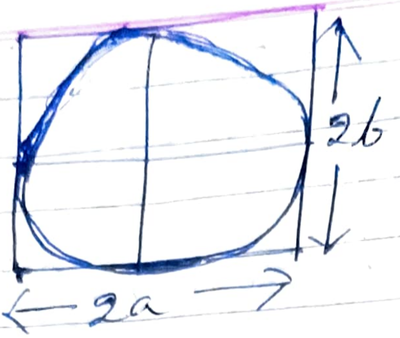


fig (4)

Case $\overline{IV}$ when $\phi = \frac{3\pi}{4}^c$ then by eqn

$$\frac{y^2}{b^2} - \frac{2xy}{ab}\cos 135^\circ + \frac{x^2}{a^2} = \sin^2 135^\circ$$

$$\frac{y^2}{b^2} - \frac{2xy}{ab}\left(-\frac{1}{\sqrt{2}}\right) + \frac{x^2}{a^2} = \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\therefore \frac{y^2}{b^2} + \frac{\sqrt{2}\,xy}{ab} + \frac{x^2}{a^2} = \frac{1}{2}$$

This eqn represent oblique ellipse lying in quadrants $\underline{II}$ & $\overline{IV}$ as shown in fig (5)

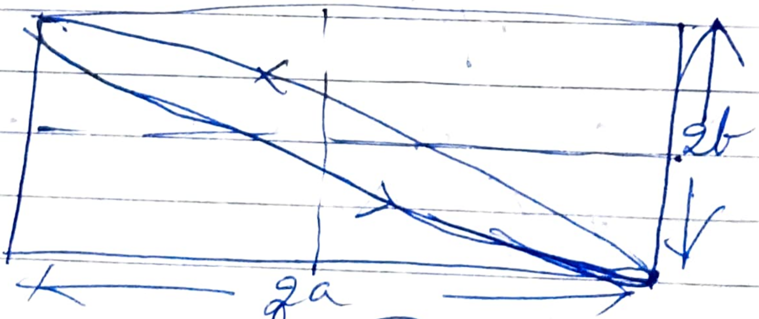


fig (5)

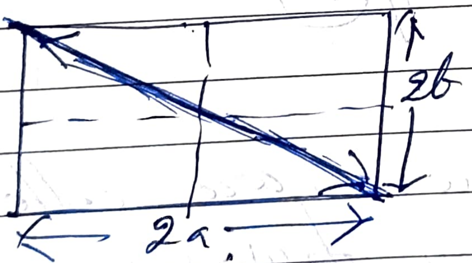
case (v) When $\phi = \pi^c$
then eqn (3) reduces to

$$\frac{y^2}{b^2} - \frac{2xy}{ab}\cos 180^\circ + \frac{x^2}{a^2} = \sin^2 180^\circ$$

$$\therefore \frac{y^2}{b^2} + \frac{2xy}{ab} + \frac{x^2}{a^2} = 0$$

$$\therefore \left(\frac{y}{b} + \frac{x}{a}\right)^2 = 0$$

This eqn represents a pair of coincident straight lines lying in quadrants II & IV as shown in fig (6)



After this, the whole cycle is represented in the reverse order as shown be

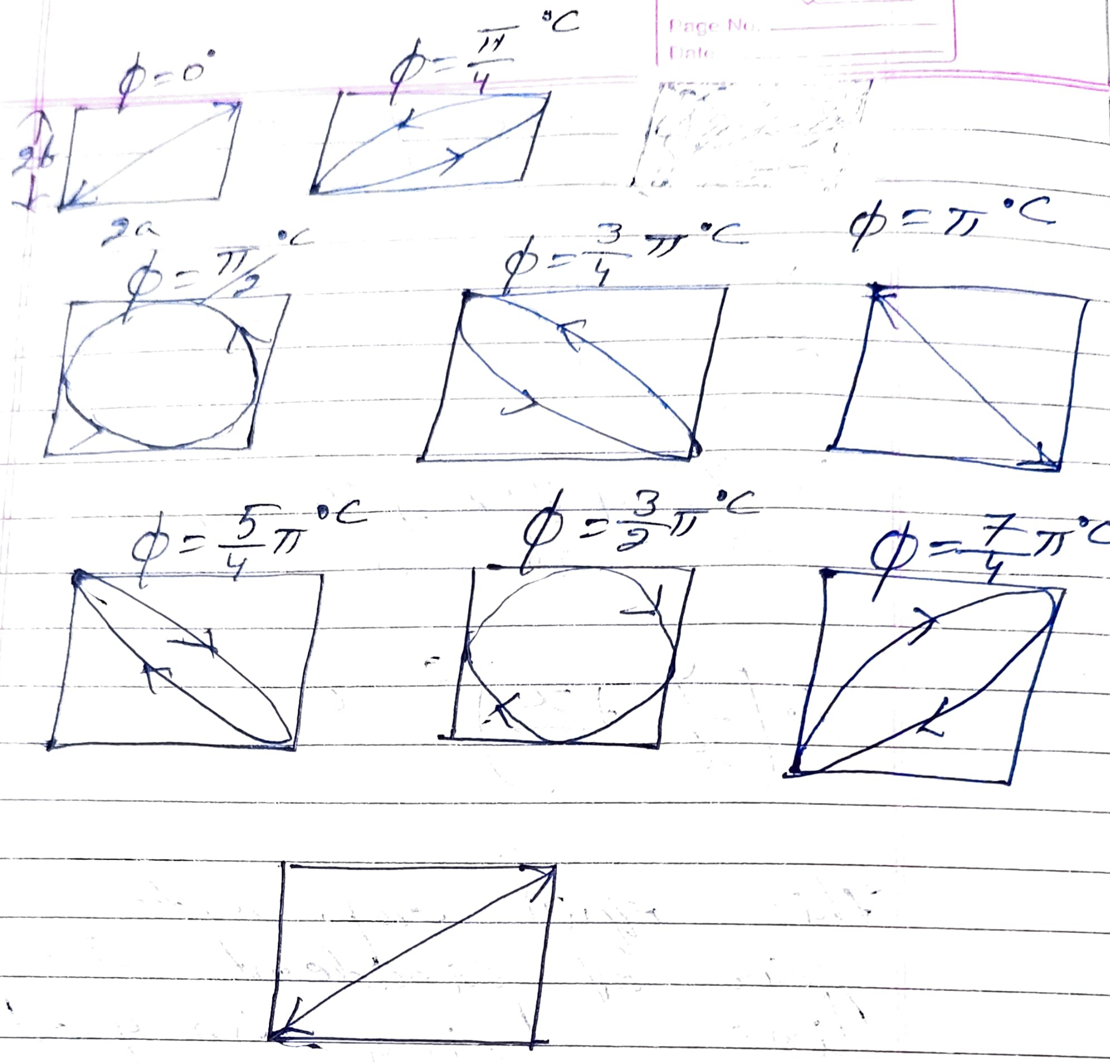


fig 7

These figures are called as Lissajous figures.

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